The hypoelliptic Laplacian in real and complex geometry

Jean-Michel Bismut
Institut de Mathématique d'Orsay

In this lecture, I will explain the construction of the hypoelliptic Laplacian in real and complex geometry, which is a deformation of the standard Hodge Laplacian.

If X is a compact Riemannian manifold, the model is the elliptic Hodge Laplacian in de Rham theory, which is written as $\square^X = \left[d^X, d^{X*}\right]$, where d^* is the L^2 adjoint of the de Rham operator d^X. If $\mathcal{X}$ is the total space of TX, the hypoelliptic Laplacian acts on the de Rham complex of $\mathcal{X}$. The symmetric bilinear form that replaces the L_2 product involves the symplectic form of $\mathcal{X}$.

I will explain why as $b \to 0$, the hypoelliptic Laplacian converges in the proper sense to the elliptic Laplacian $\square^X$. I will also describe how classical Ray-Singer analytic torsion is invariant under the hypoelliptic deformation.

Finally, I will outline the construction of the hypoelliptic deformation of the Dolbeault Hodge Laplacian.