

Index theory, Brownian motion and the hypoelliptic Laplacian

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In this talk, I will describe basic facts about the heat equation method used in the proof of the index theorem of the Dirac operator. It involves a parameter $t > 0$, which should be thought of as a temperature. To the heat equation, one can associate Brownian motion, whose dynamics is intimately related with the heat kernel.

In 1908, the physicist Langevin described a deformation of Brownian motion, associated with a mass parameter $m = b^2$. The corresponding dynamics interpolates between Brownian motion and the geodesic flow. The operator associated with this dynamics is a Fokker-Planck operator L_b .

The operator L_b is called a hypoelliptic Laplacian. It interpolates between the ordinary Laplacian and the geodesic flow. It provides a new interpolation method, in the context of Selberg's trace formula.